

# Arguments for F-Theory

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## Abstract

After a brief review of string and  $M$ -Theory we point out some deficiencies. Partly to cure them, we present several arguments for “ $F$ -Theory”, enlarging spacetime to  $(2, 10)$  signature, following the original suggestion of C. Vafa. We introduce a suggestive Supersymmetric 27-plet of particles, associated to the exceptional symmetric hermitian space  $E_6/Spin^c(10)$ . Several possible future directions, including using projective rather than metric geometry, are mentioned. We should emphasize that  $F$ -Theory is yet just a very provisional attempt, lacking clear dynamical principles.

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# 1 Introduction to M theory.

## 1.1 The beginning of String Theory.

Strings entered high-energy physics “from the back door” around 1970, as a theoretical explanation for the Veneziano formula found to fit hadron resonances. In the first period of string theory, up to 1984/5, several advances were made: fermions were included, with supersymmetry first in the worldsheet (Gervais and Sakita), then, after the Gliozzi-Scherk-Olive (GSO) projection (1977), also in spacetime (in  $10 = (1, 9)$  dimensions); string interactions were studied through vertex operators; etc.

Open strings include massless spin one particles (“ $A$ ” fields) as simple excitations, conducive of gauge theories, and closed strings (including the closed sector of open strings) exhibit also a massless spin two excitation (“ $h$ ”) which resembled the graviton; besides, a two-form (“ $B$ ”), radiated by the string, is always present, together with a scalar (“ $\phi$ ”), the dilaton, all still in the massless sector. The scale of the massive excitations is set up by the string constant (or Regge slope)  $\alpha'$ , or  $T = 1/(2\pi\alpha')$  with  $\dim T = (\text{length})^{-2}$ . By 1975 it was clear that hadron physics was better explained by QCD (Quantum ChromoDynamics), a Yang-Mills theory with  $SU(3)$  color as gauge group; also about the same time Scherk and Schwarz [1] proposed to apply string theory to gravitation and other forces because, for example, the previous  $s = 2$  and  $s = 1$  excitations had interactions, as expected, reminiscent of gravitation and gauge couplings; the proposal caught the physics community totally surprised and unconvinced.

String theory enjoyed very smooth high-energy behaviour (in particular gravitational interactions seemed to be tamed for the first time); string theory was also found to be free of anomalies (abundant in higher dimension spaces), or even better, the gauge groups and the spacetime dimensions were selected precisely by the absence of anomalies. Even the worrisome tachyons of the primitive bosonic string (living in 26 dimensions) disappeared in the GSO supersymmetric version (living in ten).

On the other hand, the Standard Model (SM) was completed also around 1975, and included just the three gauge couplings for electromagnetic, weak and strong forces. Moreover, these couplings run at different speed, to nearly coincide around  $10^{15}$  GeV [3], to reinforce the Grand Unified Theory (GUT); and this was not very far from the Planck scale,  $10^{19}$  GeV, where supposedly gravitational forces also became important. So it was reassuring that both string theory (through the extant gravitons) and the SM (through running couplings) led to schemes which foresaw gravitation close to other forces, for the first time realistically in the history of physics. For an early review of string theory see [5], and also [6] for target supersymmetry.

## 1.2 The Five Superstring Theories.

By 1985 it was clear that there were just five definite consistent superstring theories, with the following characteristics: all of them live in ten ( $= 1, 9$ ) dimensions, all were supersymmetric both on the worldsheet and in the target space, the five included gravitons, and some include gauge fields, the ones with  $N = 1$  target Supersymmetry, see below.

There is one open string with gauge group  $O(32)$  (called Type *I*), two closed  $N = 2$  superstrings, called *IIA* and *IIB*, and another two closed  $N = 1$  “heterotic” strings with gauge groups  $O(32)$  (Het-Ortho or H-O) and  $E_8 \times E_8$  (called Het-Excep or H-E), where  $E_8$  is the compact form of the last exceptional group of Cartan, with 248 generators.

The following scheme reminds the situation

$$\begin{array}{ll} IIA & \text{Het - Excep, } H - E \\ IIB & \text{Het - Ortho, } H - O \\ TypeI & \end{array} \quad (1)$$

A short description follows. We start by Type *I*; it is a theory of open (and closed) strings, with  $N = 1$  SuperSymmetry, and with a gauge group  $O(32)$ . The fundamental supersymmetry is the dimension 8 (8 transverse directions) equivalence between the vector  $\square$  and the (chiral) spinor  $\Delta_{L,R}$  representations of  $O(8)$  ( $\dim \text{spinor} = 8 = 2^{8/2-1}$ , type +1, real).

$$\begin{array}{ccc} \square & - & \Delta \\ 8 & - & 8 \end{array} \quad (2)$$

The particle content of this Type I theory in the  $m = 0$  limit includes the fundamental gauge particle (gaugeon,  $32 \cdot 31/2 = 496$  of them, plus gaugino(s)), and the closed sector content is obtained from the *skew square* of (2), to wit

$$\begin{array}{lll} (\square - \Delta) \wedge (\square - \Delta) & = & h + \phi + B - (\Psi + \psi) \\ (8 - 8) \wedge^2 & = & 35 + 1 + 28 - (56 + 8) \end{array} \quad (3)$$

(where  $\Psi$  is the gravitino field and  $\psi$  represents a spinor field). Notice the three essential ingredients, namely graviton, dilaton and two-form in the Bose sector, plus gravitino and spinors in the Fermi sector. The dilaton  $\phi$  is crucial for the perturbative expansions, because in string theory  $\exp(\phi)$  plays the role of the coupling constant  $g = g_s$ : perturbation theory is topological, and one expands in the number of “holes” in the worldsheet as a

bidimensional surface.

In the Heterotic - Orthogonal corner, which is a closed string, the particle content is the same as the closed + open sector of the previous case. On the other hand, the Heterotic-Exceptional corner has the product group  $E_8 \times E_8$  *en lieu* of the  $O(32)$  orthogonal group; notice both groups have the same dimension (496) and rank (16).

$N = 2$  Supersymmetry in target space implies closed strings only, with no gauge groups. The difference lies in the two chiralities, which can be equal (*IIB* theory, chiral) or parity transformed (*IIA* theory, nonchiral). The massless particle content is just the square of the fundamental equality (2)

$$\begin{aligned} IIA : (\square - \Delta_L) \times (\square - \Delta_R) &= h + B + \phi + A + C - (2\Psi + 2\psi) \\ &= 35 + 28 + 1 + 8 + 56 - 128 \end{aligned} \quad (4)$$

$$\begin{aligned} IIB : (\square - \Delta_{L,R}) \times (\square - \Delta_{L,R}) &= h + B + \phi + \phi' + B' + D^\pm - (2\Psi + 2\psi) \\ &= 35 + 28 + 1 + 1 + 28 + 35 - 128 \end{aligned} \quad (5)$$

(where  $D^\pm$  is the (anti-)selfdual 4-form). The bose particles in the *NS* ( Neveu-Schwarz) sector come from the bose  $\times$  bose product, whereas the *RR* (Ramond) sector is the fermi  $\times$  fermi. This distinction is very important.

It was also early noticed that the particle content of the maximal supergravity (SuGra, Sugra), namely  $11D$  Sugra [7] gives the  $m = 0$  content of the IIA string after dimensional reduction:

$$\begin{aligned} 11D \text{ SuGra} : \quad h - \Psi + C \\ 44 - 128 + 84 \end{aligned} \quad (6)$$

with the  $(D+1) \rightarrow D$  decomposition rules for the graviton  $h$  and 3-form  $C$

$$h_{D+1} = h_D + A_D + \phi_D, \quad C_{D+1} = C_D + B_D \quad (7)$$

and for the gravitino  $\Psi$

$$\begin{aligned} \Psi_{11} &= \Psi_{10,L} + \Psi_{10,R} + \psi_{10,R} + \psi_{10,L} \\ 128 &= 56 + 56 + 8 + 8 \end{aligned} \quad (8)$$

The GSW book [8] is the best reference for this period.

### 1.3 The first “Theory Of Everything, T.O.E.”

How far were we at the time from real nature? For one thing, extra dimensions ( $6 = 10 - 4$ ) supposed not to be seen in  $4D$  because the six were compact with radius of order of elementary (= Planck's) length; for another, SuperSymmetry had to be badly broken, as the

naive, “toroidal” compactification would lead to unwanted  $N = 8$  or  $N = 4$  Supersymmetry in  $4D$ . In particular, compactification and Susy breaking could be together.

Thus the Heterotic Exceptional corner of the above pentagon (1) enjoyed a temporary splendor, around 1985, as the best candidate for a realistic theory [9]: compactification in a Calabi-Yau 3-fold (i.e., a complex three dimensional Kähler and Ricci-flat manifold or orbifold) with Euler number  $\chi = \pm 6$  could be ‘close’ to generate a model of particles including even the three generations. One has to insist on  $N = 1$  only Susy in  $4D$ , as to allow parity violating couplings. It was to be hoped to get gauge forces from the descent chain of groups and subgroups

$$E_8 \times E_8 \rightarrow E_8 \text{ (+ hidden sector)} \rightarrow E_6 \quad (9)$$

as  $E_6$  is the biggest of the three best candidates for a Grand Unified Theory (GUT): these groups are  $E_6$ ,  $SO(10) = E_5$  and  $SU(5) = E_4$ : they allow for complex representations and are big enough to be simple but still encompass the groups of the Standard Model.

These early successes are well told in the book [8]; the term “Theory Of Everything” aroused around that time [10], with some opposition from more “conventional” physicists, who strongly opposed this quasi-theological approach! [11].

From the phenomenological point of view, it is also to be remarked that improved renormalization group calculations, including minimal supersymmetry, reproduce better the common intersection of the three coupling constants at somewhat higher energy, around  $10^{16}$  GeV [12].

## 1.4 M Theory.

In 1995, ten years later, three confluent developments brought about the so-called  $M$ -theory: it turned out that all five SuperString theories were related, and also with the mentioned maximal  $11D$  supergravity; there were also higher dimension extended objects ( $p$ -Branes,  $p > 1$ , with  $p = 0$  for particles,  $p = 1$  for strings, etc.), and several nonperturbative results were obtained; besides, as the theory had no adjustable parameters, it seemed to be potentially selfsufficient. The hopes for a unique theory were higher than ever; many previously skeptical physicists (including this humble reviewer) were converted to the superstring *Credo*.

These developments have to do with dualities (E. Witten [13]), membranes (P. Townsend [14]) and  $D$ - $p$ -branes (J. Polchinski [15]).

The equivalences take place mainly through the concepts of  $T$ -duality and  $S$ -duality.  $T$ -duality ([16]) is characteristic of strings, where the *winding* modes, with energy propor-

tional to the radius, exchange with *momentum* modes, with energy inverse with radius: selfduality, with enhanced symmetry, occurs at the special radius  $r = \alpha'/r$ .  $T$ -duality relates  $IIA$  and  $IIB$  theories, as the rolling interchanges chirality; it also related the two heterotic strings; recall the two groups  $E_8^2$  and  $O(32)$  share the same dimension and rank. The status of  $T$ -duality is rather robust, as it is satisfied order by order in perturbation theory.

$S$ -duality is more conjectural but more deep; it was first stated explicitly around 1990 [17], and generalized by Witten and others. It related one theory at a coupling  $g$  with another theory at coupling  $1/g$ ; it was inspired in some models in quantum field theory, mainly Coleman's proof of the equivalence of the sine-Gordon theory to the massive Thirring model with reciprocal couplings, and the Montonen-Olive conjectures about electromagnetic charge-monopole duality. Also the strong limit of strings shows the existence of "solitonic" objects, like membranes, and in general  $D-p$ -branes (Polchinski), which are endstations for open strings. In particular for a longtime a membrane was seen floating in eleven dimensional gravity, so P. Townsend in particular hypothesized, at some early date, that the  $11D$  Sugra *cum* membrane theory was just a better, eleven dimensional version of the ( $IIA$ ) string theory. The scheme was this:

$$\begin{array}{ccc}
11D \text{ } Membranes & \rightarrow & 11D \text{ } Sugra \\
?? & & dim \downarrow \text{reduc} \\
IIA \text{ } Theory & \rightarrow & IIa \text{ } Sugra \\
m=0 & & 
\end{array} \tag{10}$$

In other important prescient finding, Strominger showed in 1990 [18] that the  $7 = 10 - 3$ -dimensional dual field strength extant in  $10D$  strings was really coupled to a solitonic 5-Brane, dual to the fundamental string 1-Brane. Some of these solitonic states saturated the so-called Olive-Witten limit, and became BPS states, immune to quantum corrections; there were ideal objects for studying the strong coupling limit of several (eventually, all) string theories.

The strong ( $S$ ) coupling limit of the five superstring theories could then be described:

- i) The  $IIB$  theory dualizes to itself: the role of the dilaton/ $RR$  axion scalars and the fundamental/ $RR$  2-forms are crucial to show this; besides, the theory exhibits *D-branes* as the sources of the  $RR$  even-dimensional  $p$ -forms: in fact, there are branes in all odd dimensions up to 9. The strong duality group was conjectured to be the infinite discrete modular group  $SL(2, \mathbb{Z})$ , which will play an important role in the first version (Vafa [19]) of *F*-Theory (see below).

ii) The Type *I* theory is *S*-dual to the Het-Ortho theory (Witten-Polchinski [20]): recall both have the same full spectrum.

iii) But the two big surprises were the relations of strings with  $11D$  Sugra: The strong limit of the *IIA* theory in the mass zero regime gives exactly the particle content of  $11D$  Sugra, as shown by Witten (March 1995, [13]); this was really the crucial starting paper in *M*-theory; but the name, “*M*”, where *M* stands for Membrane, Magic or Mystery according to taste, was not endorsed until October, 1995 ([21]). In other words: the  $11D$  Sugra theory compactified in a large circle becomes the massless (strong) limit of the *IIA* theory, and the extant  $11D$  membrane shrinks to the  $10D$  string by double dimensional reduction.

iv)  $11D$  Sugra compactified in a segment  $D^1 = S^1/Z_2$  is very likely equivalent to the Het-Excep theory (Witten and Hořava [22]); namely the boundary is acceptable only supporting gauge groups, indeed one  $E_8$  group at each corner of the segment; now as the segment shrinks, and the 11 dimensions become ten, we reproduce the Het-Excep theory; at least, as the authors emphasized, *if* there is a limit of the  $11D$  Sugra/segment construct, *then* it has to be the  $m = 0$  sector of the H-E theory. In another development, Witten ([23]) argued that if the segment is much larger than the Calabi-Yau space (which links the H-E corner with the  $4D$  world), the universe appears fivedimensional, and one can even make the dimensionless gravitational constant  $G_N E^2$  coincide with the GUT value ( $\approx 1/24$ ) of the other three.

Branes can appear as solitons as well as sources for the *RR* forms, in the same way that the fundamental (or *F*) string is the source for the *NS* 2-form. For example  $11D$  Sugra supports the membrane (Townsend) and *also* a dual 5-Brane [24]: duality for branes occur at the field strength level; e.g. the  $11D$  membrane ( $p = 2$  extended object) couples to the 3-form *C* of before, and it is its field strength, a 4-form, which dualizes to a 7-form, supported by a 5-brane. And in *IIB* theory, the *RR* radiation forms, the 0-, 2- and 4-(selfdual) potential forms require as *RR* charges or sources Polchinski’s *D-p*-Branes, which are extended objects acting as terminals of open strings.

There is a further equivalence, between *IIB* and Type *I*; it is achieved through the so-called  $\Omega$  (parity) projection, forming unoriented strings. For dualities, Branes and *M*-theory the reader should consult the books [25] and [26], and the magnificent collection of original papers gathered in [27].

The full set of relations between the different types of Superstring Theory together with  $11D$  Supergravity are schematized in the famous hexagon

*11D Sugra*

$$\begin{array}{ll}
 IIA & H - E \\
 \\ 
 IIB & H - O \\
 \\ 
 TypeI &
 \end{array} \tag{11}$$

## 2 Some difficulties with *M*-theory.

### 2.1 Some developments in *M*-theory.

*M*-theory has not lived up its expectations. The theory is so badly defined, that it is even difficult to say it is wrong! In the elapsed ten years, it has not advanced very much. At the begining of it, ca. 1996, many new avenues were explored, by Witten and others, with no real progress; we just quote some of them:

- 1) Matrix models [28]
- 2) AdS/CFT correspondence [29]
- 3) Noncommutative geometry [30]
- 4) F-theory [19], which we shall develop below in Sect. 3.
- 5) K-Theory [31]
- 6)  $E_{11}$  symmetry [37]
- 7) Topological strings and twistors [32]

In “*M*-theory as a Matrix model” [28] the authors suggest a precise equivalence between 11*D* *M*-theory and the color  $N = \infty$  limit of a supersymmetric matrix mechanics describing particles (as *D*-( $p = 0$ )-Branes).

The Anti de Sitter - Conformal Field Theory correspondence (AdS/CFT; [29], see also [33]), is an important development, independently of its future use in a “final” theory. In the spirit of the Holographic Principle of ‘t Hooft and Susskind, it is argued that the large  $N$  ( $N$  colors) limit of certain  $U(N)$  conformal field theories includes a sector describing supergravity in the product of Anti-de Sitter spacetime and a sphere: the light cone where the CFT sits can be seen as the boundary of the AdS space. In particular, restriction of strings/*M*-theory to Anti de Sitter spacetime is dual to superconformal field theories.

Roughly speaking, concentrated gravity in  $D+1$  dimensions exhibit the degrees of freedom in the light cone with  $D$  dimensions, because the “bulk” lies inside the black hole horizon, and therefore it is unobservable.

Noncommutative geometry is a new advance in pure mathematics due to A. Connes [34], [35]; it is interesting that position or momentum operators commutation rules became nontrivial in presence of magnetic fields or the 2-forms  $B$  of string theory. This is the initial thrust of the long Seiberg-Witten paper, [30].

$K$ -theory as applied to  $M$ -theory [31] considers the three-form  $C$  as a kind of local “connection”, and it might be that  $C$  as a 3-form is not globally defined; the gauge group is the group  $E_8$ , which seems to play an important role in  $M$ -theory (but it is explicit only in the H-E corner). The right mathematical tool to deal with this situation is the so called  $K$ -Theory, which classifies vector bundles. See also [36].

P. West has developed an interesting scheme [37] based in the infinite Lie algebra called  $E_{11}$ ; it is a triple extension of  $E_8$ , beyond the affine and hyperbolic Kac-Moody cases. For a recent treatment see [38].

For a recent review of topological strings see [39]

## 2.2 Stringy Microscopic Black Hole Entropy.

Perhaps the only “success” of  $M$ -theory so far is the accurate counting of microstates, and hence of the entropy, in some types of Black Holes (BH) by Strominger and Vafa [40].

In gravitation the equivalence principle forces the existence of a horizon for the field of a point mass (the same phenomenon does not occur for a point charge inspite the same  $r^{-2}$  force law). In quantum mechanics the BH radiates [41], and it has therefore temperature and entropy: the later is the horizon area/4, in Planck’s units. The above authors were able to compute this as related to the number of microstates, as one should do of course in the statistical mechanics interpretation of thermodynamics.

Let us mention that recently Hawking [42] has withdrawn his previous claim that there is information loss in BH radiation: there might be subtle correlations in the black hole temperature radiation, “remembering” what came into the black hole in the first place.

## 2.3 Difficulties with $M$ -theory.

Inspite of these separate advances (and some others we omit) the theory languished, both for lack of new stimulus, as for being unable to complete its deficiencies. Among the un-

satisfactory features of  $M$ -theory as first established we can quote:

- 1) There is no clear origin for the  $IIB$  theory: indeed, the relation within the hexagon is by means of  $T$ -duality, which means going to nine (or less) dimensions; one should hope to obtain the proper  $IIB$  theory in  $10D$  in some limit of the (future)  $M$ -theory.
- 2) Even the Heterotic Exceptional corner of the hexagon is a bit far fetched if related to the  $11D$  Sugra: where do the two  $E_8$  groups come from? As we said, Witten and Hořava were careful enough to state “IF the  $11D$ /Segment reproduces a string theory, THEN it has to be the H-E corner”. Of course, the two gauge groups  $E_8$  at boundaries are forced by anomaly cancelation, but one does not “see” them directly in the  $M$ -theory.

This argument is really more powerful: it is only the  $IIA$  corner which fits reasonably well with the  $11D$  theory; indeed, neither the  $H-O$  theory nor the Type  $I$  come directly from  $M$ -theory, one has to recur to  $T$ -duality; also, even as regards the  $IIA$  theory, see point 5) below.

3) There is lack of a dynamical principle; in this sense string theory, at least, is more conventional than  $M$ -theory: excited strings can be treated, at least perturbatively, as a Quantum Theory; however, membranes and higher ( $p > 2$ ) Branes have no known quantization scheme. This is related to the dimensionless character of the 2-dim quantum fields, which allows very general background couplings, and this is simply not true for membranes etc. A related argument is this: for particles and strings, the geodesic problem (minimal volume) in the “Polyakov form” is equivalent to gravitation in one or two dimensions (Weyl invariance is needed in the string case). However, this is no longer true from membranes onward: although there is no really graviton degrees of freedom in  $3D$ , gravitation is “conic”, and presents definite phenomena.

4) The maximal natural gauge group in  $M$ -theory, in its  $11D$  version, seems to be the  $\binom{8}{2} = 28$ -dim orthogonal  $O(8)$  group, generated by the 28 massless gauge fields down to  $4D$ . But this group is too small, and unable to accomodate the minimal GUT group of the standard model,  $O(10)$  ( $SU(5)$  is insufficient to account for massive neutrinos); there are possible way outs, like composite fields,  $SU(8)$  as gauge group, etc., but none really very convincing [43].

5) There is the so-called massive  $IIA$  theory [44] which again does not fit well with  $M$ -theory. It is a new version of the  $IIa$  nonchiral supergravity in ten dimensions, in which the two-form  $B$  “eats” the one-form  $A$  (the vector field) and grows massive: it is exactly the Higgs mechanism one degree further; but then the analogy with the reduced sugra in  $11D$  no longer subsists, and hence it corresponds to no clear corner in the  $M$ -theory hexagon.

## 3 Forward with *F*-theory.

### 3.1 The Original Argument.

Cumrum Vafa seems to have been the first, back in February 1996 [19], to advertise an “*F*” Theory, in 12 dimensions with  $(-2, +10)$  signature, after the first introduction of the *M*-theory by Witten in March, 1995 (although the name, *M*-theory, was given a bit later), and as an extension of the same; for another early hint on  $12D$  space see [45].

The argument of Vafa was related to the *IIB* superstring theory; as it lives in  $10D$ , it cannot come directly from *M*-theory in 11 dimensions: the two possible one-dimensional compactifications from  $11D$  were on a circle  $S^1$ , giving the *IIA* theory, and in a segment  $D^1$ , giving the Het-Excep theory; besides, there were no questions of any strong coupling limit: as we said, the *IIB* string was a case in which selfduality under  $g_s \rightarrow 1/g_s$  was proposed, because the two scalars (dilaton and axion) make up a complex field  $z$ , which transform homographically under the discrete residue of  $SL(2, R)$ , a well known invariance group (although noncompact!) of *IIB* Sugra; the two *B* fields (fundamental and RR) also transform naturally under this  $SL(2, R)$  group; Townsend was the first to propose then that a discrete  $SL(2, Z)$  subgroup remained, and it was then obvious that the duality included inversion of the string coupling,  $g_s \rightarrow 1/g_s$ , where  $g_s$  is the exponential of the vev of the dilaton  $\phi$ .

The point of Vafa was that the group  $SL(2, Z)$ , the so-called modular group, was the moduli group for a torus: the inequivalent conformal structures in the torus are labeled by the modular group. And Vafa, of course, interpreted this torus, with metric  $(-1, +1)$ , as a compactifying space from a  $(-2, 10)$  signature space in twelve dimensions; the name “*F*-theory” was proposed by Vafa himself, meaning probably “father” or “fundamental”; the argument for increasing one time direction is subtle and we shall show it more clearly later below. So the idea is that the *IIB* theory on space  $\mathcal{M}_{10}$  comes from an elliptic fibration with fiber a 2-torus, in a certain  $12D$  space: symbolically

$$T^2 \rightarrow \mathcal{V}_{12} \rightarrow \mathcal{M}_{10} \tag{12}$$

Then, *F*-theory on  $\mathcal{V}_{12}$  is equivalent to *IIB* on  $\mathcal{M}_{10}$ .

### 3.2 Dynamical Arguments for *F*-theory.

There are several other arguments, mainly aesthetic or of completion, in favour of this *F*-theory. We express all of them rather succinctly, as none are really thoroughly convincing. However, there are so many that we believe taken together they give some force to the idea of a 12-dimensional space with two times; for the particle content in this space see a proposal later.

1) *IIB* theory really comes from 12 dimensions, with toroidal compactification; this was the original argument of C. Vafa [19]. The space  $\mathcal{V}_{12}$  admits an elliptic fibration, and the quotient is the frame for the *IIB* theory. Most of compactifications from strings ( $10D$ ) or *M*-theory (in  $11D$ ) can be carried out from  $12D$  *F*-theory [19], [46].

2) There is a Chern-Simons (CS) term in the  $11d$  Sugra lagrangian, together with the conventional kinetic terms for the graviton, gravitino and 3-form  $C$ , and another “Pauli type” coupling, see e.g. [7]

$$\mathcal{L} = \dots + C \wedge dC \wedge dC + \dots \quad (13)$$

Now a CS term can be understood as a boundary term, hence claiming for an extra dimension interpretation  $(dC)^3$  [47]. This favours the interpretation of the group  $E_8$  as a gauge group in *M*-theory [36].

3) There is a  $(2, 2)$  Brane extant in the Brane Scan, once one allows for some relax in supersymmetry dimension counting. In fact, in the brane scan of Townsend, if one insists on  $(1, D - 1)$  signature, one finds the four series of extended objects (and their duals), ending up with the  $11D$   $p = 2$  membrane [48]. By relaxing the signature, but still insisting in supersymmetry (in the sense of bose-fermi matching), one encounters a few new corners [49]; Susy algebras in this most general context were already considered in [50]. Indeed, there is a  $(2, 2)$  membrane living in  $(2, 10)$  space. The membrane itself was studied carefully in [51]; by doubly dimension reduction, this  $(2, 2)$  membrane supposes to give rise to the string in *IIB*-theory.

4) The algebra of 32 supercharges of  $11D$  Sugra still operates in  $12 = (2, 10)$  dimensions, with the (anti-)commutation relations

$$\{Q, Q\} = 2 - form + 6^\pm - form \quad (14)$$

so  $\dim Q = 2^{12/2 - 1}$  and  $528 = \binom{12}{2} + \binom{12}{6}/2$  (for the interpretation see later).

Now, in  $11D$ ,  $\dim Q = 2^{(11-1)/2} = 32$  (type +1, real), and the superalgebra is

$$\{Q, Q\} = P_\mu + Z^{(2)} + Z^{(5)} \quad 32 \cdot 33/2 = 11 + 55 + 462 \quad (15)$$

namely, 11-dim translations plus a two-form and a 5-form, understood as central charges. But the  $11D$  superalgebra clearly comes from the simpler 12-dim algebra of above: the 2-form gives 1-form and 2-form, and the selfdual 6-form gives rise to the five-form. The

signature must be  $(2, 10)$ , which gives  $0 \bmod 8$  for the type of  $Q$ ; in case of  $(1, 11)$  signature is  $2 \bmod 8$ : the charges would be complex, or  $2 \cdot 32 = 64$  real: twelve dimensional two-times space is maximal for 32 real supercharges.

On the other hand, the direct reduction from  $12D$  space to  $10D$  via a  $(1, 1)$  torus would yield the  $IIB$  string from the  $(2, 2)$  membrane (as noted above) and the self-four form  $D^\pm$  from the selfdual 6-form.

5) We have remarkable relations in dimensions  $8D$ ,  $9D$ ,  $10D$ : effective dimensions of a  $(1, 9)$  string theory,  $M$ -theory in  $(1, 10)$  and  $F$ -theory in  $(2, 10)$  as regards supersymmetry. The fundamental supersymmetry extant in 8 effective dimensions in string theory is  $8_v - 8_s$  between the vector ( $v$ ) and the spinor ( $s$ ), and squaring

$$(8_v - 8_s) \cdot (8_v - 8_{s'}) = h - \Psi + C = 44 - 128 + 84 \quad (16)$$

That is, the square fits in the content of  $11d$  Sugra with 9 effective dimensions.

But another square  $\#(h - \Psi + C)^2 = 2^{15} - 2^{15}$  gives a 27-plet in  $D_{eff} = 10$  (or  $(2, 10) = 12D$ ) as we shall see later, because this is related to our proposal for the particle content of  $F$ -theory. The fact that the square of the fundamental irreps of Susy in 8 effective dimensions fits nicely in irreps of  $9D$ , and the square again fits in irreps of a effective  $10D$  theory, is most notorious and unique, it is certainly related to octonion algebra, and it was first noticed by I. Bars in [58] before the  $M$ -theory revolution.

6) The content of  $11D$  Sugra is related to the symmetric space (Moufang projective plane over the octonions)

$$OP^2 = F_4/Spin(9) \quad (17)$$

where  $O(9)$  is the massless little group in  $11D$ ; this was shown by Kostant [56]. There is a natural extension by complexification [52] to the space

$$OP_C^2 = E_6/Spin^c(10) \quad (18)$$

related naturally to the  $12 = (2, 10)$  space as  $Spin^c = Spin(10) \times_{1/2} U(1)$ , where  $O(10) \times O(2)$  is the maximal compact group of the  $O(2, 10)$  tangent space group in  $12D$ .

We shall explain this in detail in section four. It will be enough to remark here that now the candidate GUT group is  $O(10)$ , and there is no problem with fitting the SM group ( $U(3, 2, 1)$ ) within it.

7) Compactification from 11 dimensions to 4 is preferable through a manifold of  $G_2$  holonomy in order to preserve just  $N = 1$  Susy in  $4D$ ;  $G_2$  is a case of  $7D$  exceptional

holonomy, the only other being  $Spin(7)$ , acting in 8 dimensions, very suitable for our  $12 \rightarrow 4$  descent (this was already noticed by [19]); again, the  $11D$  case generalizes naturally and uniquely to 12; and we have the nice split  $12 = 4 + 2 \cdot 4$ . Indeed, it seems that an argument like 4) can be also made here. Trouble is, we really need 8-dim manifolds with  $(1, 7)$  signature and exceptional holonomy, which are not yet fully studied.

### 3.3 Numerical Arguments.

These are really “numerological”, and we include them for lack of a better reasoning, and also for some kind of “completeness”.

1)  $O(2, 10)$ , the tangent space group, is the largest group of the Cartan coincidences; namely, it is

$$O(2, 10) = Sp(2, Oct) \quad (19)$$

in a definite sense that we can briefly comment: namely the groups  $Sp(1, 2; K)$  where  $K$  are the reals, complex, quaternions or octonions, are equivalent to some Spin groups. This is part of the so called Cartan identities [54]; the simplest of these is  $Sp(1, R) = Spin(1, 2)$ , and the largest is  $Sp(2, Oct) = Spin(2, 10)$ , pertinent to  $F$ -theory.

2) In the  $12 = 4 + 8$  split, all spaces are even, and even dimensional spaces have integer dimensions for bosons and halfinteger for fermions, which seems to be the natural thing; for odd dim spaces, it is the other way around.

3)  $78 = \dim E_6 = \dim (\text{Poincar\'e or (A)dS } (2, 10)) = \dim O(13) = \dim Sp(6)$  is the only dimension, after the  $A_1 = B_1 = C_1$  identities, with triple coincidences of group dimensions; does this hint towards a relation between  $E_6$ , as a GUT group, with the  $(2, 10)$  space of  $F$ -theory?

### 3.4 Unusual characteristics of $F$ -Theory.

Actually,  $F$ -theory as presented here is very different from the usual theories; in particular, as the  $\{Q, Q\}$  (anti)commutator does not contain the translations, there is no “Poincar\'e” algebra, and consequently there is no direct concept of mass as eigenstate of momentum, and in particular no massless limit.

How does one understand this? In any arbitrary (pseudo-)riemannian manifold the only general structural feature is the structure group of the tangent bundle, or in physical terms the tangent space group, which is the (pseudo-)orthogonal group; in our case of the  $(2, 10)$  space, it is  $O(2, 10)$ ; so it is gratifying that indeed, the supercharge algebra (14) gives just

this generator, as a two-form. The concrete shape of the space is left unanswered, for the moment.

As for the selfdual 6-form in (14), it is hoped it will be related to the matter content, in the same sense as the central charges in 11D Supergravity are related to membranes. Is it possible to relate this selfdual 6-form to the extant (2, 2)-Brane?

How do we incorporate two times in a theory of physics, in which the arrow of time is so characteristic? At face value, there are two ways out: either one of the times compactifies, so possible violations of causality are of the order of Planck's length, or there is a gauge freedom to dispose of one of the times; I. Bars [59] favours the second, but there are also consistent schemes with the first alternative.

## 4 Euler Multiplets.

### 4.1 Euler Triplet as the 11d Sugra particle content.

In 1999 P. Ramond realized [55] that many irreducible representations (irreps) of the  $O(9)$  group, including the Sugra irrep

$$h - \Psi + C = 44 - 128 + 84 \quad (20)$$

i.e. graviton, gravitino, 3-form, can be grouped in triplets with many coincidences, besides the “susy” aspect (all but one of the Casimirs correspond, etc.). The underlying mathematics was cleared up by B. Kostant [56], and goes with the name of “Euler triplets” (later multiplets). It is related to the symmetric space

$$OP^2 = F_4/O(9) \quad (21)$$

which defines the octonionic (or Moufang) plane. Namely the spin irrep of the orthogonal group acting in  $R^n$  with  $n = \dim F_4 - \dim O(9) = 16$ , splits in 3 irreps of  $O(9)$  because the Euler number of projective planes is three:  $\chi(OP^2) = 3$ ; it is supersymmetric because the Dirac operator has index zero, and this measures the mismatch between left- and right-nullspinors.

We shall not delve in the proofs, only on the construction. Let  $G \supset H$  be a pair: Lie group and subgroup, of the same rank, with  $G$  semisimple and  $H$  reductive (i.e.  $H$  can have abelian factors). The  $Pin(n) = \Delta_L + \Delta_R$  group in the difference of dimensions  $n = \dim G - \dim H$  suggests a Dirac operator  $D$ : left spinors to right spinors (these manifolds are even-dimensional, have  $w_2 = 0$  or are  $Spin^c$ , so they admit spinor fields). Then  $\ker D$  and  $\text{coker } D$  split each in representations of  $H$ , with a total of  $r = \chi(G/H)$

irreps of  $H$ . Notice  $\dim Pin(n) = 2^{n/2}$  and  $\dim \Delta_{R,L} = 2^{n/2-1}$ . Now this Euler number can be computed from the Weyl group of the  $(G, H)$  pair:

$$\chi(G/H) = \frac{\# \text{Weyl group of } G}{\# \text{Id. of } H} \quad (22)$$

So there is a kind of supersymmetry associated to some homogeneous spaces. The matching is as follows:

$$\Delta_L - \Delta_R = \sum_{i=1}^{\chi} \pm D_i(H) \quad (23)$$

$$2^{n/2-1} - 2^{n/2-1} = \sum_{i=1}^{\chi} \pm \dim D_i(H) \quad (24)$$

In fact, each representation of the higher group  $G$  generates a similar multiplet of  $\chi$  irreps of  $H$ ; the above construction corresponds to the Id irrep of  $G$ ; see [56].

We shall use a complexification of the Moufang plane as a proposal space for the particle content in  $F$ -theory; before this we give simple examples of the Ramond-Kostant construction.

## 4.2 Some examples of Euler multiplets.

There are many example of this Ramond-Kostant construction; we shall exemplify only two of them. Consider the complex projective space

$$CP^n = SU(n+1)/U(n); \quad (25)$$

the homology of  $CP^n$  is  $(1, 0, 1, 0, 1, \dots)$ , so  $\chi(CP^n) = n+1$ ; on the other hand,  $\dim \Delta_{L,R}(2n) = 2^{n-1}$ . The “supersymmetric partition” is e.g. for  $n=4$

$$\Delta_L - \Delta_R = [1^0]^{+2} - [1]^{+1} + [1^2]^0 - [1^3]^{-1} + [1^4]^{-2} \quad (26)$$

(or  $8-8=1-4+6-4+1$ ), with the usual convention:  $[D]^q$  is the irrep of  $U(n)$  in the form  $[D]$ , an irrep of  $SU(n)$ , and  $q$ , a label for the  $U(1)$  factor, where  $U(n) = SU(n) \times_{/n} U(1)$ . In eq. (26) only the fully antisymmetric irreps  $[1^r]$  enter, and  $[1]$ ,  $\dim = 4$ , is the fundamental irrep of  $SU(n=4)$ . Notice for  $n$  odd “Supersymmetry” is just Poincaré duality.

Another more sophisticated example is this [57]. Take  $G = SO(10)$  and  $H = SO(8) \times SO(2)$ . Then  $\dim G/H = 16$  and Euler  $\# = 10$ . The dimensions of the split multiplet is

$$1^2 - 8^{3/2} + 28^1 - 56^{1/2} + (35^{+0} + 35^{-0}) - 56^{-1/2} + 28^{-1} + 8^{-3/2} + 1^{-2} \quad (27)$$

### 4.3 The proposal.

The original discovery of Ramond was related to the Sugra  $11D$  multiplet, associated to the Moufang plane (this association was first seen by Kostant).

Now Atiyah and Berndt [52] have shown that the first row of groups in the so-called magic square

$$O(3) \quad U(3) \quad Sp(3) \quad F_4 \quad (28)$$

associated to projective planes (e.g.  $RP^2 = O(3)/O(1) \times O(2)$ , etc.) can be complexified in a precise mathematical form; thus the second row is  $U(3), U(3)^2, U(6)$  and  $E_6$ ; the complexified symmetric spaces are also well studied; in particular the fourth is

$$OP^2 \rightarrow OP_C^2, \quad F_4/SO(9) \rightarrow E_6/SO(10) \times SO(2) \quad (29)$$

It is natural to extend the Sugra multiplet, associated to  $O(9)$  as the massless little group of the Poincaré group in  $(1, 10)$ , to the multiplet associated to the complexification of the Moufang plane, namely

$$Y := \frac{E_6}{SO(10) \times SO(2)} \quad (30)$$

This is a remarkable space: it is symmetric, hermitian, and exceptional; the subgroup should really be  $Spin(10) \times_{/2} U(1) = Spin^c(10)$

How do we connect this with  $F$ -theory? Because  $O(10) \times O(2)$  is the maximal compact subgroup of the tangent group  $O(2, 10)$  in our  $12D$   $F$ -theory, as we said above! In ordinary,  $(1, D-1)$  spaces, the massive group is  $O(D-1)$  and the massless  $O(D-2)$ . But here, with a  $(2, D-2)$  space, we suggest naturally to take the maximal compact subgroup, namely  $O(2) \times O(D-2)$ .

The corresponding multiplet of particles for  $Y$  is huge: the Euler number is 27,

$$\chi(OP_C^2) = \#Weyl(E_6)/\#Weyl(O(10)) = 51840/1920 = 27 \quad (31)$$

and the split  $\Delta_L(32) - \Delta_R(32)$  is  $2^{15} - 2^{15}$  or  $32768 - 32768$ .

The particle content has been calculated by I. Bars [58]. Because the complex nature of  $Spin(10)$ ,  $\dim \Delta(10) = 16$ , it helps to consider the chain

$$Spin(8) \subset Spin(9) \subset Spin(10) \subset SU(16) \subset SO(32) \quad (32)$$

and it is given in the following Tables. (We omit the  $q$  label associated to the abelian part  $SO(2)$ , and write only half of the Table, 9 out of the 17 entries, to be completed by duality,

$O(32)$			$\Delta_L$	$2^{15}$	
$SU(16)$	$\text{Id}=[1^0]$	$[1^2]$	$[1^4]$	$[1^6]$	$[1^8]$
$SO(10)$	scalar	3-form	Weyl + 1050	4312+3693	4125+8085+660
dim	1	120	1820	8008	12870

Table 1: Putative Bose Particle content of  $F$ -Theory

$O(32)$			$\Delta_R$	$2^{15}$
$SU(16)$	$16=[1]$	$[1^3]$	$[1^5]$	$[1^7]$
$SO(10)$	spinor	h-g-ino	672+3696	8800+2640
dim	16	560	4368	11440

Table 2: Putative Fermi Particle content of  $F$ -Theory

as  $\binom{n}{k} = \binom{n}{n-k}$ ).

This is a huge multiplet; notice it contains an scalar, a spinor, a 3-form, the Weyl tensor  $[2^2]_H$ , and the hypergravitino (h-g-ino),  $\Psi_{\mu\nu\alpha}$ , plus more exotic representations. It does not contain gravitons nor gravitinos. There is no clear-cut between massive and massless particles. Evidently, there should be a way to cut down the number of states in order to obtain some sensible physics; we have no idea how this can be done, out of the simple remark that in the standard minimal supersymmetric model (with no gravitons) the number of states is  $128 - 128$ , the square root of those obtained here. Perhaps a square root *different* of that obtaining the  $11D$  content from the  $12D$  is singled out.

## 5 Outlook.

What will be the physics of this putative  $F$ -theory? If true, it will be a theory very different from the usual ones. We focus here in one particular aspect, the projective spaces  $OP^2$  and  $OP_C^2$ ; notice the particle content of the second does contain neither the graviton nor the gravitino; could this be an indication that projective geometry will take the place of the riemannian (metric)? After all, the second is a particular case of the first; also, much of the dynamics in  $M$ -theory, which we hope will persist in  $F$ -theory, has to do with Branes: intersection, pile-up,  $D$ -type, etc. This reminds one of the operations in projective geometry, with inclusion and intersection of subspaces; we take as a hint the fact that both the Sugra triplet and the 27-plet considered here single out projective spaces.

Even the usual lagrangian formalism used so far becomes suspect: dualities in  $M$ -theory make two different lagrangians to produce the same physics, which is indicative of a more general formalism, one of the forms of which is lagrangian: it is like the theory of transformations in quantum mechanics, which is the invariant formalism, whereas the coordinate

or momentum representations are just partial aspects.

We finish by making the obvious comparison with the times of the Old Quantum Theory, 1913-25: there were some results, some recipes, but absence of a full dynamical theory, which came only with the matrix mechanics of Heisenberg in 1925. Perhaps something of this kind of surprise is in store for us. It is in this spirit that we offer these somewhat wild speculations on the space manifold and particle content of the future theory.

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